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Principal Examiner Feedback

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Pearson Edexcel International GCSE

in Further Pure Mathematics (4PM0)

Paper 01

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Introduction

Candidates found paper 1 somewhat more difficult than paper 2. The reasons for this are not immediately apparent though many experienced problems with the first three questions on paper 1. This was surprising in the case of questions 1 and 2 as these tested topics which are also on the various GCSE/iGCSE specifications. Question 3 was a topic that always causes all but the best candidates problems. The recent timetable alteration, which gives a gap of over a week rather than a couple of days between the two papers, may have contributed as candidates had extra time for further revision and could concentrate on the topics which had not been tested on paper 1.

Candidates are becoming more confident in working with radians although some still prefer degrees and fail to change their answers into radians. Rounding seems to be less of an issue too although there are still cases of candidates either failing to round at all or truncating instead of rounding. Inequalities gave problems with either the incorrect inequality used through including (or excluding) 0 or reversing the inequality sign.

Paper 1

Question 1

This question proved to be a demanding start to the paper for some candidates.

(a) Most candidates attempted this part although there were varying degrees of success. Many found it difficult to complete the square on $\mathrm{f}(x)$ because of the negative coefficient of x^2. A number of candidates rewrote the given function as $2x^2 - 5x - 6$ or attempted to complete the square to give $-2\left(x \pm \frac{5}{2}\right)^2 + k$ resulting in M0A0A0. Only a minority of candidates gained full marks in this part.

(b) Despite the difficulties in completing the square on $\mathrm{f}(x)$, the majority of candidates were able to identify the maximum value of $\mathrm{f}(x)$ from their expression, and the value of x for which this maximum occurs, achieving both B follow through marks. There were a number of candidates who used calculus in this part, and this was generally a very successful method usually resulting in both B marks being awarded.

(c) The most common approach here was to use calculus. Most candidates successfully found the correct value of x, although a small minority then forgot to substitute this value back into $\mathrm{g}(x)$ to find the maximum value of $\mathrm{g}(x)$ which was necessary for the award of the M mark. Achieving $x = \sqrt[3]{\frac{5}{4}}$ only, resulted in M0A0B1. Although a few candidates did recognise the similarity between $\mathrm{g}(x)$ and $\mathrm{f}(x)$ they usually attempted to complete the square on $\mathrm{g}(x)$ mostly unsuccessfully. The most efficient method (seen only very rarely) was to recognise that $\mathrm{g}(x)=\mathrm{f}(x^3)$, allowing candidates to write down the maximum value of $\mathrm{g}(x)$ and the value of x for which this occurs with no working.

Question 2

(a) This part of the question was accessible to all with the majority achieving both marks. Only the very weakest candidates were unable to draw the straight-line graphs correctly. A range of methods was used, some drawing tables of values, some plotting 2 or 3 points, or some finding the intersections with the coordinate axes. A few were able to draw the graphs with no working shown.

(b) Many also achieved both marks here, although some did not realise they should draw the line $y=-1$ and used the x-axis to define the region instead. The majority of candidates shaded the correct enclosed region. The most common incorrect shading of the region was $3x+2y\le 12$, $y\ge 3x-3$, $y\ge -1$ and $x\ge 0$.

The majority of candidates were able to score all four available marks in parts (a) and (b).

(c) Many students did not attempt part (c) at all. Of those who did, most identified the coordinates of all 3 points of intersection by reading them directly from the graph. There was little evidence of solving simultaneous equations to find the accurate coordinates, although apart from the point (2, 3) which could be read exactly from the graph, we allowed x = 0.6, 0.7, 0.8 for $x=\frac{2}{3}$, and x = 4.6, 4.7 or 4.8 for $x=\frac{14}{3}$ for the M marks, these being the two intersections with the line $y=-1$.

For the required vertex for the maximum value of P, the correct exact answer of $P=\frac{59}{3}$ was seen occasionally, although we also allowed P = 19.7, but it was rare to see full marks awarded in this part.

Question 3

There were many correct solutions to this question. Those that fell by the wayside did so for three main reasons.

(1) There was a small minority of candidates who could not start the question because they did not realise that application of chain rule was required.

(2) Some candidates left both h and r in their expression for V, and differentiated either with respect to r or h. The M mark could not be available for such attempts.

It was of course also possible to work in terms of r only, but this involved a more complicated chain rule as well the necessity of also finding $\frac{\mathrm{d}h}{\mathrm{d}r}$ and there were no successful attempts seen using this method.

These two groups of candidates gained at most one mark for stating a correct chain rule.

(3) A significant number of candidates saw that V had to be expressed in terms of h, but put $1.5r = h$. They were usually able to differentiate this dimensionally correct expression without error, and if this was the only mistake they scored all 3 method marks. This pattern was easy to spot as it lead to the often seen answer of $\frac{\mathrm{d}h}{\mathrm{d}t} = 1.21$.

It was pleasing to note that there were hardly any rounding errors in giving the final answer to 3 significant figures.

<u>Question 4</u>

For the most part this question was answered very well indeed.

(a) The vast majority of candidates found the correct solution of $t = 3$ and $t = 5$. There were very few errors when using factorisation or the formula, and a few were able to write the two values of t without working. The latter gained full credit as the quadratic was very simple to solve. A small minority attempted to find the stationary points by differentiating the given v.

(b) Most candidates were just as successful in this part substituting their mostly correct values for t into a correctly differentiated function. The odd error seen arose from an incorrectly differentiated expression, or a few did not substitute both values of t to find the acceleration at both of the times found in (a). Avery small number candidates stated that an acceleration of -4 m/s^2 was not possible, and we could not ignore subsequent working for this as there was an error in understanding. It was pleasing to that virtually no candidates integrated to find the solution to this part of the question.

(c) This part of the question caused the most confusion for candidates. Most correctly found the integral but some failed to substitute $t = 3$ into their integrated expression. A substitution of $t = 5$ was frequently seen showing that the question had not been read carefully, but substitutions of $t = 4$ or even $t = -4$ were also seen clearly highlighting a lack of understanding. There was only one M mark in this part, and so the method had to be complete including the substitution of the correct value of t. Otherwise, the only available mark was the B mark to find the value of the constant of integration.
The most common successful method was to find the indefinite integral, calculate the value of c and then substitute $t = 3$ into the equation.
Those candidates who attempted to calculate the definite integral between $t = 3$ and $t = 0$ arrived at a value of s = 36. It was clear that many candidates who attempted this method did not understand the implications of the starting point and the need to subtract 4 from their answer. Another error seen (shown below), which was observed in several scripts highlighted a lack of understanding of the constant of integration and the starting point of P.

$$t = 0, s = -4 \quad \therefore \int_0^3 \left(2t^2 - 16t + 30 - 4\right)\, \mathrm{d}t = \left[\frac{2t^3}{3} - 8t^2 + 30t - 4t\right]_0^3$$

Overall however, this question was answered well by candidates with most scoring at least the first five marks.

Question 5

(a) This part of the question were well-answered by virtually every candidate. The majority successfully found the required values for the table but a not-insignificant minority were penalised for rounding the y value of 3.33 incorrectly (the question explicitly stated two decimal places where appropriate).

(b) The plotting of points was generally accurate, although a surprising number of candidates appeared to struggle with the scale on the grid provided, despite it being straightforward. A number of candidates attempted to join the points with straight line segments, whereas a smooth curve was required. For those few candidates who had made errors in calculating the y coordinate or in plotting their points, the resultant very irregular curve should have been an indication to check their work in part (a).

(c) This part of the question caused problems for many candidates, and fewer than half were able to re-arrange $y=\dfrac{x^3+2}{x+1}$ into the form $y=ax+b$. Many candidates realised that $(x+1)$ had to be a factor of the resulting quadratic but then did not take this sufficiently far to gain the first M mark.

A number of candidates used $y=\dfrac{-x^2+3x+4}{x+1}$ to generate some coordinates and found the correct straight line $(y=4-x)$ this way. Full credit was given for these efforts when they were correct.

Most candidates who successfully deduced the correct line went on to easily find the required value of x, but a significant number gave a value that was 'too accurate' (the question specified one decimal place) - perhaps suggesting they had found the value on their graphic calculators, and thus lost the final A mark unless the correct rounded value of $x = 1.6$ was seen.

Because the question clearly states ‘By drawing a suitable straight line on your graph’ he correct root in the given range of $x = 1.6$ without the correct straight line drawn received no marks at all in this part.

Question 6

(a) A surprising number of candidates could not start this part of the question at all, and many of those who did attempted to used cosine rule which of course got them nowhere. Others simply stated $\theta=\tan^{-1}\sqrt{255}=86.4...^\circ$ hence $\cos 86.4^\circ=\dfrac{1}{16}$ which again received no marks given that the question specifically stated ‘without finding the value of θ'.

There were two correct approaches, both using Pythagoras theorem, but one considerably simpler than the other.

The expected approach was to use the fact that in a right angle triangle, the opposite side was of length $\sqrt{255}$ and the adjacent side of length 1, and using Pythagoras theorem, deduce that the hypotenuse was 16, hence $\cos\theta = \frac{1}{16}$.

The other correct method seen was; $\tan^2\theta = 255 \Rightarrow \frac{\sin^2\theta}{\cos^2\theta} = 255 \Rightarrow \sin^2\theta = 255\cos^2\theta$

$\Rightarrow 1-\cos^2\theta = 255\cos^2\theta \Rightarrow \cos^2\theta = \frac{1}{256} \Rightarrow \cos\theta = \frac{1}{16}$

(b) Virtually every candidate knew that to find an angle given three sides, cosine rule is required and almost all earned the first two marks in this part using either $\cos\theta = \frac{1}{16}$ or for the first A mark, we accepted use of cos 86.4°. Using cos 86.4 invariably lost the second A mark because the length of *x* is exact. There was some clumsy algebra and the required length of 8 cm was only seen in about half of the total number of scripts. Applying cosine rule resulted in a three-term quadratic equation in *x*. Many candidates nowadays use their calculators to solve quadratics. This is perfectly acceptable, and if the correct value of *x* was achieved following the correct three-term quadratic, full credit was given with or without working in the form of factorisation or applying the formula. If, however, the incorrect value of *x* was achieved (with or without the correct three-term quadratic), and no method to solve the quadratic was written down, examiners could not be sure if a correct method was used and therefore no credit was given. Candidates must always show all working as stated in the rubric in the 4th bullet point on the front page of the paper.

(c) Those candidates who got this far in the question used mainly sine rule (usually correctly) to find the value of $\angle ABC$, with the incorrect value of *x* from part (b) followed through for the M mark.

(d) The area of the triangle was also easily found by candidates who found a value for *x*, and as in part (c), any incorrect value was followed through for the M mark.

Question 7

(a) The usual error in this part arose from a lack of understanding of how to deal with the $-4x^2$. In order to gain the method mark, the power of *x* must be correct, together with the correct binomial denominator. It was not uncommon for candidates to simply use *x* or more commonly just $4x^2$. The latter error was noted even in the work of very able candidates who went on to score full marks in the rest of the question. When a correct substitution was seen by examiners this did not necessarily guarantee that candidates would arrive at the correct expression. Errors in dealing with negatives, or even the simple arithmetic to evaluate a term were frequently seen.

(b) It was clear in this part that a number of candidates did not know how to process the fact that the required expansion included x^2.

(c) The vast majority of candidates had a clear understanding of what was required and it was common for candidates to achieve at least both method if they had made a computational error in part (a). The biggest issue observed in this part actually hindered candidates in part (d). It was very common for candidates to arrive at the correct solution in this part, albeit in the incorrect order (which did not matter at all), and then reorder the terms into ascending powers of *x*. However crucially for part (d), they would miss a term in their rewrite. They of course achieved full credit in part (c) as we ignore subsequent errors if they do not contradict earlier work.

(d) A very small number of candidates completely misunderstood the question and tried to divide $(3+x)$ by either their binomial expansion of $\left(1-4x^2\right)^{-\frac{1}{2}}$ or even by $\left(1-4x^2\right)^{-\frac{1}{2}}$ itself. The marks in this part depend on candidates' achievements earlier in this question. It was not uncommon for candidates to achieve the method marks only due to previous errors in this question. There were some careless errors within the integration of a correct response to part (c), whereby the most common error was seen when integrating $6x^2$. This was frequently seen to be integrated to $3x^3$ rather than $2x^3$. Centres should advise their students to show full working as $\frac{6x^3}{3}$ will always gain full credit rather than 'doing it in your head' and making a trivial error causing the loss of two A marks in this case; one for an erroneous integrated expression, and one for the inevitably incorrect final value. As in previous years, a very small number of candidates showed a lack of understanding of how to process this question which led to some very unusual integrals and subsequent final answers. It is beneficial to also remind candidates about the importance of showing their working when substituting values into their integrated expressions as a number of candidates had integrated correctly unfortunately arrived at the incorrect solution. The lack of evidence to support a correct substitution resulted in the loss of both the M and A mark here.

Question 8

(a) The majority of candidates attempted to solve the correct equation here, but some did not realise (or read the question carefully which clearly stated; Find the **two** possible values of *r*), that the equation $r^4 = 4$ should have 2 solutions and so could not be awarded any marks, even when they had found the positive root correctly. However, this was generally well answered although throughout we were a little surprised that many candidates did not identify, or use the fact that $\sqrt[4]{4} = \sqrt{2}$ The value of $\sqrt[4]{4}$ was of course given full credit and candidates generally went on to use either form correctly in later parts.

(b) Most candidates used the correct definitions to obtain a correct equation, although a small number attempted to use summation formula instead. This was well-answered but too many candidates attempted to use an inexact form of their answer from (a) for which they could not reach the exact value of 3 and also could not be awarded the first A1 either as this required a correct, exact equation.

(c) This was well answered with most candidates remembering the correct summation formula and only a small number attempting to use 9 instead of 10 when substituting in for n. There were many valiant efforts here despite having obtained incorrect values of a and r but this could have provided a further opportunity to identify a sign error in their earlier working when attempting to substitute a negative a or r despite the wording in the question specifying that it was positive from this point.

(d) The majority of candidates setup a correct inequality and substituted their values correctly. However, many candidates confused *n* for *n*-1 and as such could not be awarded any marks in this part.
Log work for the second M1 was generally accurate, although some numerical errors such as also considering $(n-1)\log\left(3\times\sqrt{2}\right)$ following a correct statement in the line before, caused some candidates to fall astray. Many candidates reached 20.287... and then either failed to write this as an integer (as *n* can only be an integer) or incorrectly wrote 20. The inequality signs were in virtually every case used correctly.

Question 9

(a) The majority of candidate recognised what was required of them in this question. Most successfully wrote down the correct quadratic equation in part (a). The common errors were to either write their equation with fractional coefficient of *x* $\left(\text{from } \alpha+\beta=-\dfrac{5}{2}\right)$, and/or to write an expression, missing the = 0 in their final answer.

(b) It was clear that many candidates knew the equivalent expressions for $\alpha^2+\beta^2$ and $\alpha^3+\beta^3$. The latter sometimes contained algebraic errors so M1A1M0A0A0 was a common marking pattern because the M mark was awarded for correct algebra only. However, candidates who were able to start with the correct algebraic expression for $\alpha^3+\beta^3$ usually achieved 5 out of 5 marks. It is worth noting that we still continue to see $(\alpha+\beta)^2=\alpha^2+\beta^2$ and $(\alpha+\beta)^3=\alpha^3+\beta^3$ very occasionally.

(c) Candidates clearly knew they had to find the sum and product of the given roots in such a form to use their answers from part (b). However the accurate algebraic manipulation required was beyond a number of candidates. Most of the errors were careless slips with signs or powers and thorough checking through of their working would have avoided this. For example, a few candidates found they were needing to substitute a value of $\alpha-\beta$ or $\alpha^2-\beta^2$. This should have been a warning to go back and check algebra. More candidates were successful in finding the correct product of the roots rather than the sum of the roots. If a candidate was able to gain the first two M marks in (c) they would usually always also gain the A marks. Regardless of the first four marks, most candidates were able to successfully use their product and sum to correctly form a quadratic equation, giving the final M mark. Candidates who had achieved the first four marks in usually went on to achieve 6 out of 6 marks with = 0 only occasionally omitted.

Question 10

This was a long question carrying 16 marks, a real challenge for candidates , and with the exception of part (a) was a clear discriminator of ability.

(a) This part of the question was usually efficiently done, with roughly equal numbers starting $\cos^2\theta$ and $\frac{1}{2}(\cos 2\theta + 1)$. The majority of candidates scored all three marks here.

(b) This part however, was beyond the compass of many. Most attempts started from $8\cos^4\theta + 8\sin^2\theta - 7$, and certainly the majority of the relatively few fully correct responses were in this category. What nearly all these correct responses had in common was that $\sin^2\theta$ was converted into $(1-\cos^2\theta)$ at the beginning, so that subsequent work consisted essentially of applying, one way or the other, the given result of $\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$.

Correct responses starting from $\cos 4\theta$ were very rare, and usually avoided introducing sine ratios except to substitute for $\cos^2\theta$ until the end. Again, the basic process of starting with cos4θ was applying $\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$ twice to cosine ratios but in the opposite direction.

(c) Quite a few candidates saw that what was required here was to manipulate the given identity in (b) $[\cos 4\theta = 8\cos^4\theta + 8\sin^2\theta - 7]$ into $\cos 4\left(\theta - \frac{\pi}{6}\right) = \frac{1}{2}$. However, it was not uncommon to then see $\cos\theta = \frac{1}{2}$ or $\cos 4\theta = \frac{1}{2}$. Of all the letters (roman or greek) available to use as a substitution for $4\left(\theta - \frac{\pi}{6}\right)$, θ was not the wisest choice. Having used this substitution, very few reverted back $4\left(\theta - \frac{\pi}{6}\right)$ and there were many answers giving just $\frac{\pi}{12}$, which is coincidentally one of the two correct solutions, but achieved through an incorrect method and therefore could not receive credit. Responses giving both values of θ were very rare. The one mostly given was only $\theta = \frac{\pi}{12} + \frac{\pi}{6} = \frac{\pi}{4}$ and the missing solution was $\theta = -\frac{\pi}{12} + \frac{\pi}{6} = \frac{\pi}{12}$. For several of the most able candidates, this was the only mark dropped in the entire question.

(d) For those candidates who got this far in the question, as long as they could see that they needed to re-write the expression using the given result from (b), full marks often followed. However, most candidates stopped short of even attempting it. As in part (d) of question 7, candidates are very strongly advised to show a full substitution of both $\frac{\pi}{2}$ **and** 0 because without the correct exact answer seen, examiners cannot be sure that a correct substitution is being applied, with the loss of the final M mark as well as the A mark.